Methods for Analyzing Change Over Time

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Overview

- What are longitudinal studies?
- Some designs
- Considerations for selection of data analytic methods
- Review several methods and compare
- Focus on specific group of methods

What are Longitudinal Data?

- Data arising from repeated measures of the same variable at a number of different time points
- Observations can be for a single subject (i.e. Time Series) or multiple subjects, single or multiple variables
- Longitudinal vs. repeated measures: the function of time

Why Conduct Longitudinal Studies?

- When change is the object of study: only way to investigate change is with repeated observations
- "Temporal ordering of events is often the closest we get to causality" (van der Kamp & Bijleveld, 1998)

Designs for Longitudinal Studies:

(from Bijleveld et al., 1998)

- Simultaneous cross-sectional: different samples (e.g., 4th, 5th & 6th graders) measured on same variables for 1 wave only.
- Trend study: new, comparable sample at each new time of measurement, same variables over time
- Time-Series: same subjects followed for successive time points--many occasions
- Panel study: Whole sample retained in a study and measured repeatedly--fewer occasions (i.e., 3 - 5).
- Longitudinal Panel study: Multiple cohorts measured at multiple occasions

How Many Observations?

- "We speak of longitudinal data whenever we have observed more than once." (van der Kamp & Bijleveld, 1998)
- Traditional longitudinal design: pre/posttest
- "2 observations do not a longitudinal study make" (Rogosa, 1991)

Considerations for model selection

- Level of measurement for the DV
- Change Parameters: appropriate for question?
  - Fixed vs. Random or Mixed Effects
  - Modeling Inter- and Intra-individual Differences
- Parameter Estimation Procedures
- Missing Data

Level of Measurement of the Dependent Variable

Level of Measurement for DV

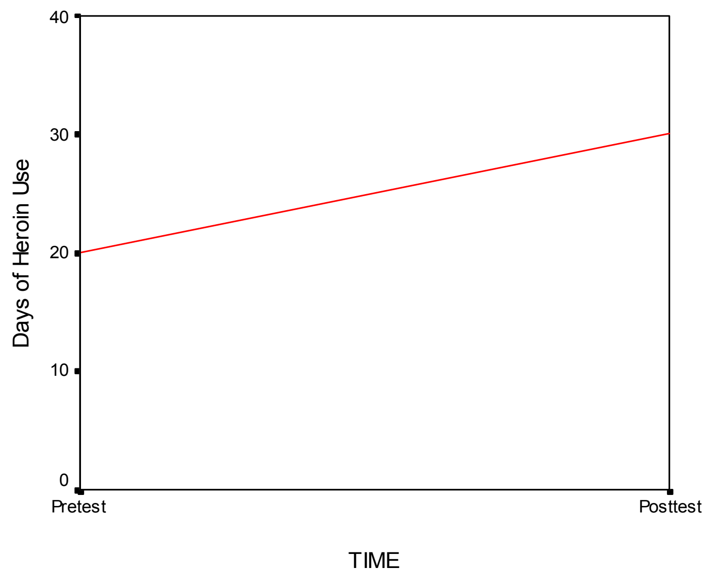
- Continuous outcomes: e.g., depression scores, days drug-free, math achievement
- Dichotomous outcomes: reoffender vs. non-; cancer vs. no cancer, pass vs. fail
- Ordinal outcomes: e.g., Likert scale responses (e.g., 0="poor health" to 5 = "excellent health")

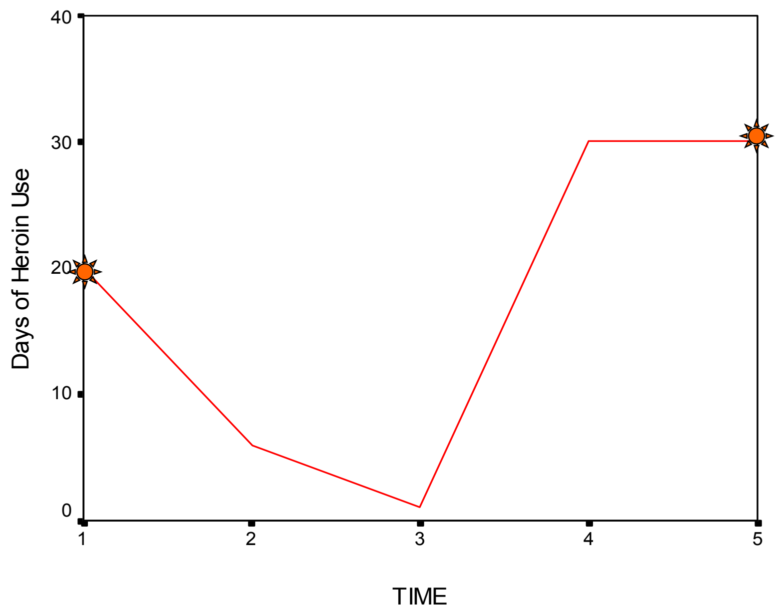
Change parameters: Appropriate to questions regarding change?

Fixed vs. random effects
Inter- vs. Intra-individual change

The Nature of Change

- Change as a discrete event: the difference between 'before' and 'after'
- The problem with difference scores (Rogosa, 1995)
  - 2 points: straight line
  - Estimates *amount* of change
  - Ltd info on *pattern* of change, scatter in data

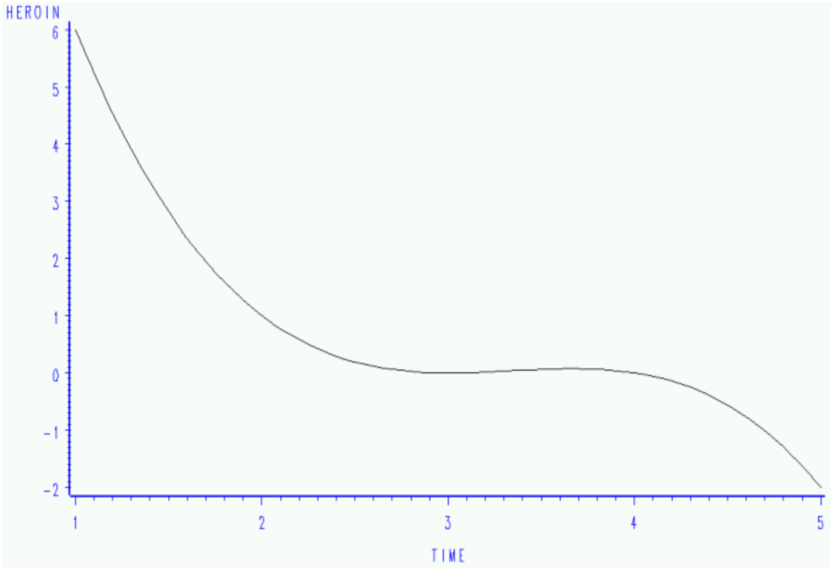




Rate of Change

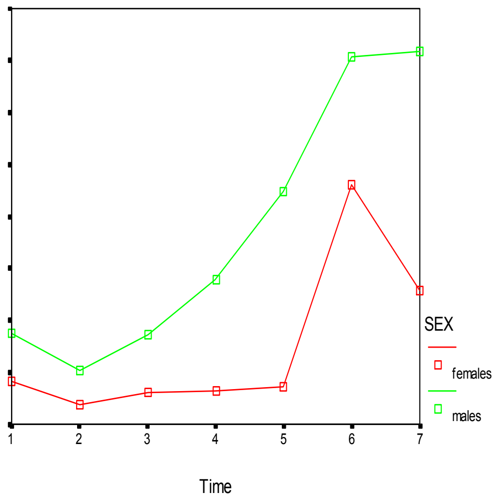
- How fast, slow, steady, jagged, flat, etc.
- Change is a continuous process, has a trajectory that can be modeled using a mathematical function
- Mathematical terms to describe change: Onset, escalation, acceleration, deceleration, plateau

Figure 1. Third Order Polynomial Growth Function



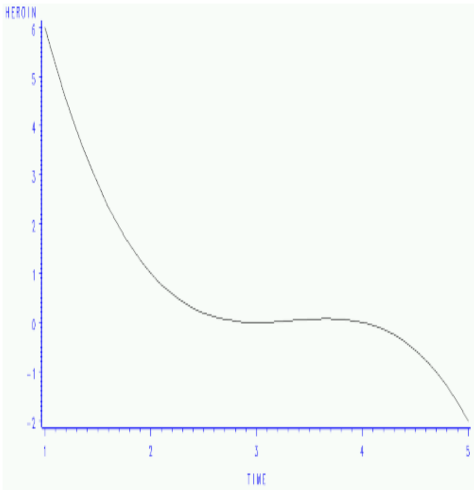
Change Parameters
Traditional Fixed Effects Models

- Change is a discrete event
- Change = difference score
- Function-free models
- Pre/post ANCOVA
- Repeated Measures ANOVA



Change Parameters
Individual Growth Models

- Change is a continuous process
- Function-based models
- Intercept = baseline status estimate
- Slope = rate of change in Y
- Latent Growth models, MLMs, slopes-as-outcomes



Fixed vs. Random Effects

- Fixed effects: all possible levels of the factor in which researcher is interested are present in the study (e.g., dosage)
- Random effects: levels of the factor present in the study assumed to be random sample from a universe of relevant levels (e.g., schools)

Change: Fixed or Random effect?

- Fixed: all time points in which we're interested are sampled in the study
  - Generalization limited to time points sampled
  - Effects are constant, without measmt error
- Random: time points are sampled from a universe of all relevant time points
  - Generalizations extend beyond time points sampled
  - Effects assumed to reflect measurement error

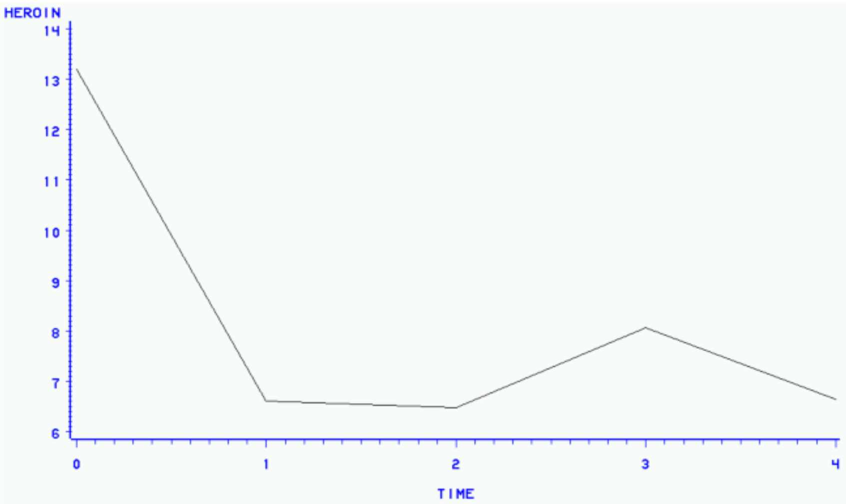
Fixed vs. Random Coefficients

- Applies to linear model parameters, e.g., intercept & slope
- Traditionally assumed to be fixed, estimated from the data
  - Obtain mean solution
- Random coefficients: values assumed to be distributed as probability function
  - Obtain mean solution
  - Individuals allowed to deviate from mean solution (measured as variance parameter)

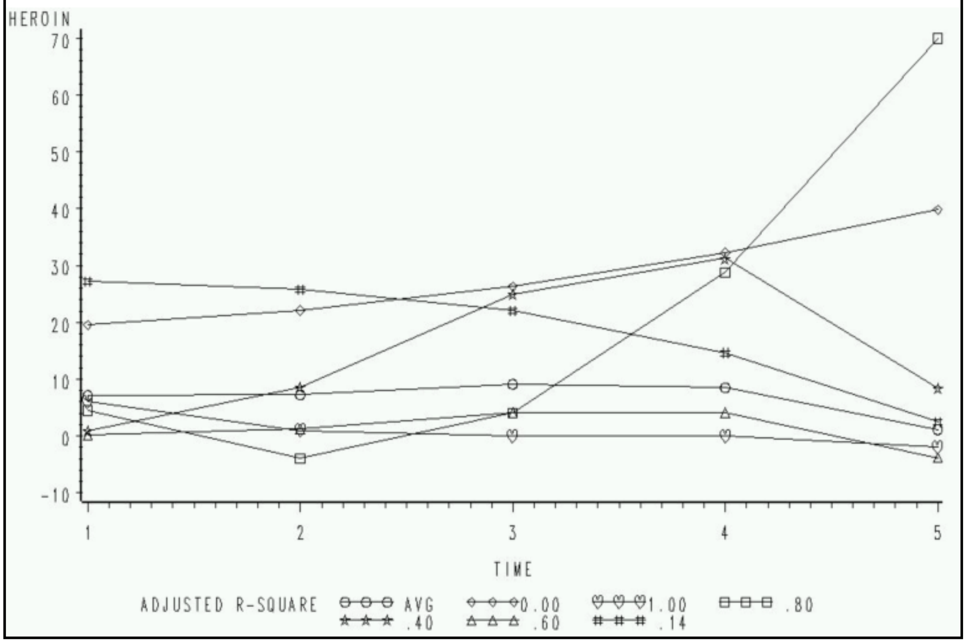
Change: Fixed or Random?

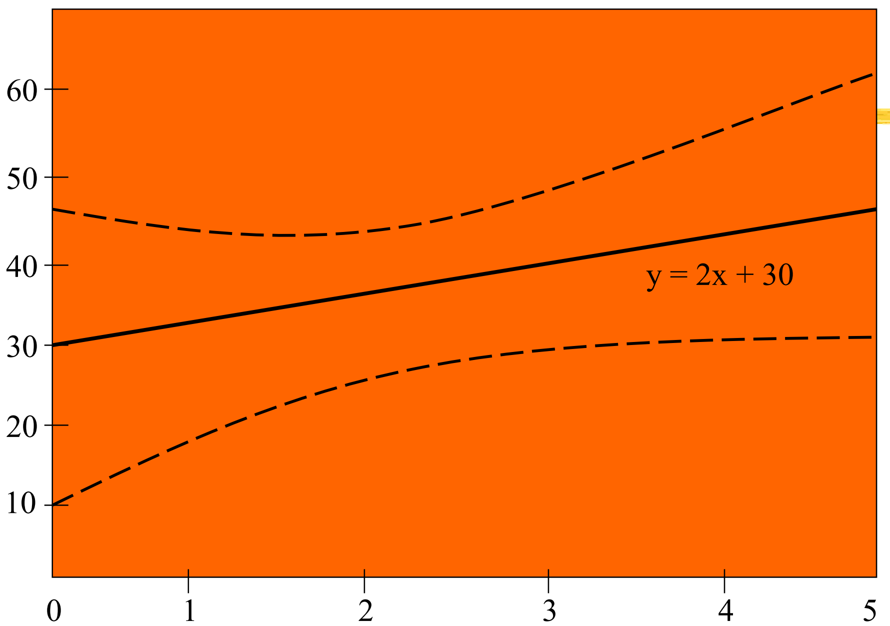
- Fixed coefficient models: assume everybody changes at the same rate
- Random coefficient models: allows for individual differences & explaining them

Fixed Effect: The Average Trajectory



Random Effect: Individual Trajectories





Fixed vs. Random Coefficients:
Modeling Individual Differences

- Ordinary linear regression: assumes fixed intercept and slope
- Fixed: mean solution, deviations considered unexplained error
- Values estimated from data

Traditional fixed coefficients

$$y_{ij} = \underbrace{a}_{\text{intercept}} + \underbrace{b}_{\text{slope}}\underbrace{x_{ij}}_{\text{predictor}} + \underbrace{\varepsilon_{ij}}_{\text{residual}}$$

outcome intercept slope predictor residual

a = everyone has the same starting point (intercept)
b = everyone changes at the same rate (slope)

Fixed coefficients output

Coefficients[a]

Model		Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	56.592	.453		125.044	.000
	Grade plus 1	.079	.133	.015	.595	.552

a. Dependent Variable: Math achievement

$$\text{Mathach} = \underbrace{56.59}_{\text{Initial status}} + \underbrace{.08}_{\text{Rate of change}}(\text{time})$$

Cont'd

- Random coefficients models: multilevel models, latent growth models
- Parameter values assumed to be distributed as a probability function
- Allow individuals and groups to deviate from the mean solution (e.g., avg. slope)
- Estimate more parameters: mean solution + variance about the means

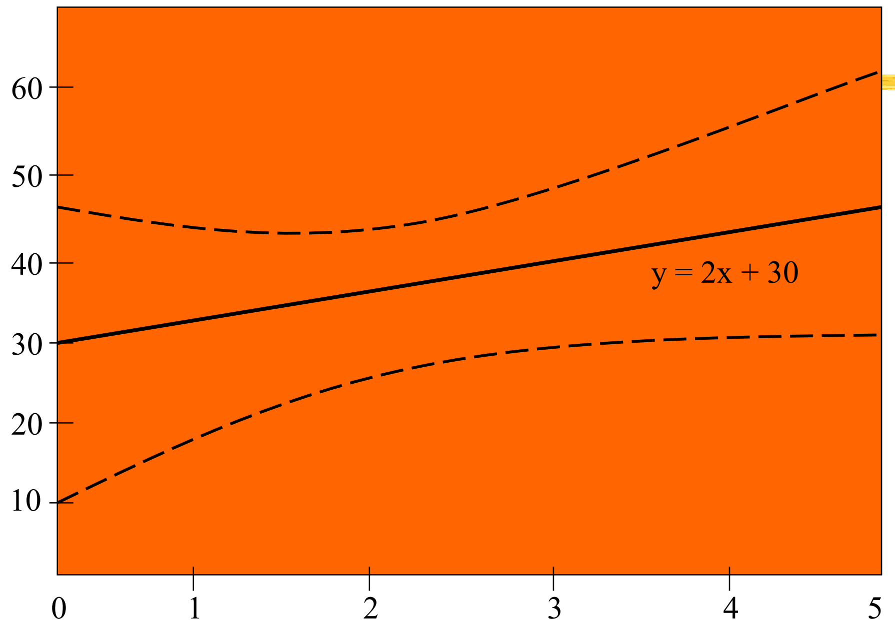
Random coefficients

$$y_{ij} = \pi_{0j} + \pi_{1j} x_{ij} + \varepsilon_{ij}$$

outcome intercept slope predictor residual

π_{0j} = individuals vary on starting point (intercept)
π_{1j} = individuals vary on rate of change (slope)

Random coefficient solution with random intercept and slope



The limits of fixed effects models of change

- *developmental theories typically do not posit change in terms of time-specific comparisons (e.g., females are expected to be higher than males on a certain attribute at Time 2 above and beyond their previous level of standing relative to the group mean at Time 1). Instead, developmental theory tends to construe change as a continuous growth process over time, and this process is described in terms such as individual differences in onset, escalation, acceleration, plateau, and deceleration*

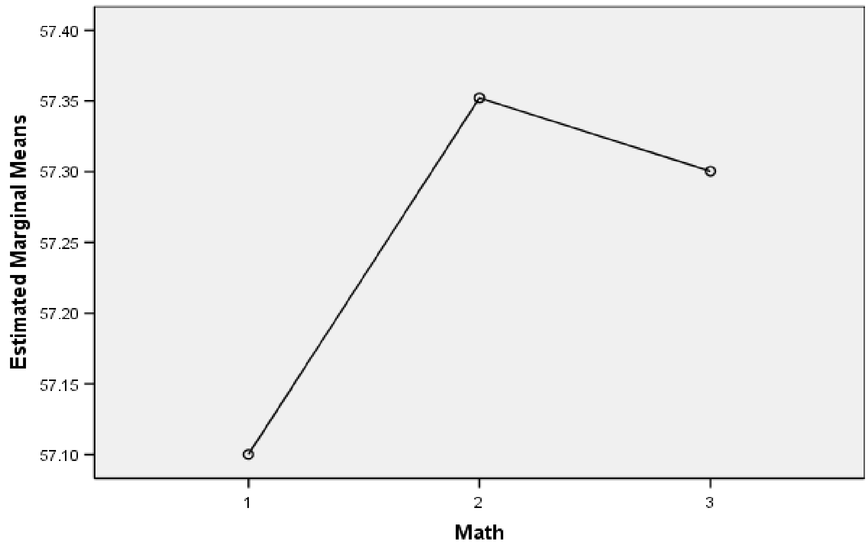
  -- (Curran and Muthen, 1996, p. 5)

Inter- vs. Intra-individual differences in change

Inter- vs. Intra-individual change

- Interindividual change = between groups
- Intra-individual change = within subjects
- Traditional fixed effects models: focus on between group differences
- Individual growth curve methods: focus on both individual and group level differences in change

Repeated Measures ANOVA: Inter-individual change
Within-subjects = Avg change over time



Repeated Measures ANOVA Inter-individual Change: Between-subjects effects

Estimates

Measure: MEASURE_1

Gender	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Male	58.458[a]	.465	57.544	59.372
Female	56.044[a]	.425	55.209	56.880

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.

Pairwise Comparisons

Measure: MEASURE_1

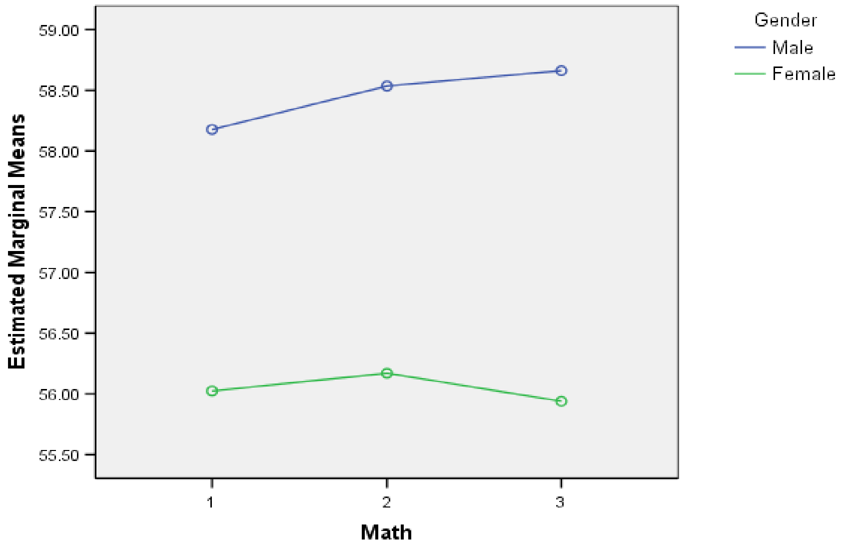
(I) Gender	(J) Gender	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	95% Confidence Interval for Difference[a] Upper Bound
Male	Female	2.414*	.632	.000	1.172	3.655
Female	Male	-2.414*	.632	.000	-3.655	-1.172

Based on estimated marginal means

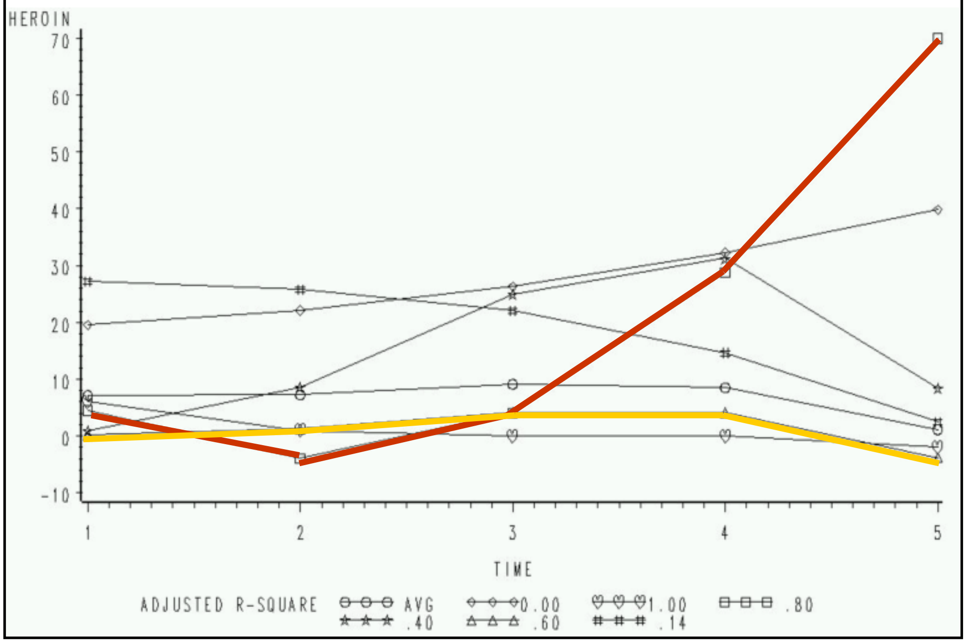
*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Least Significant Difference (equivalent to no adjustments).

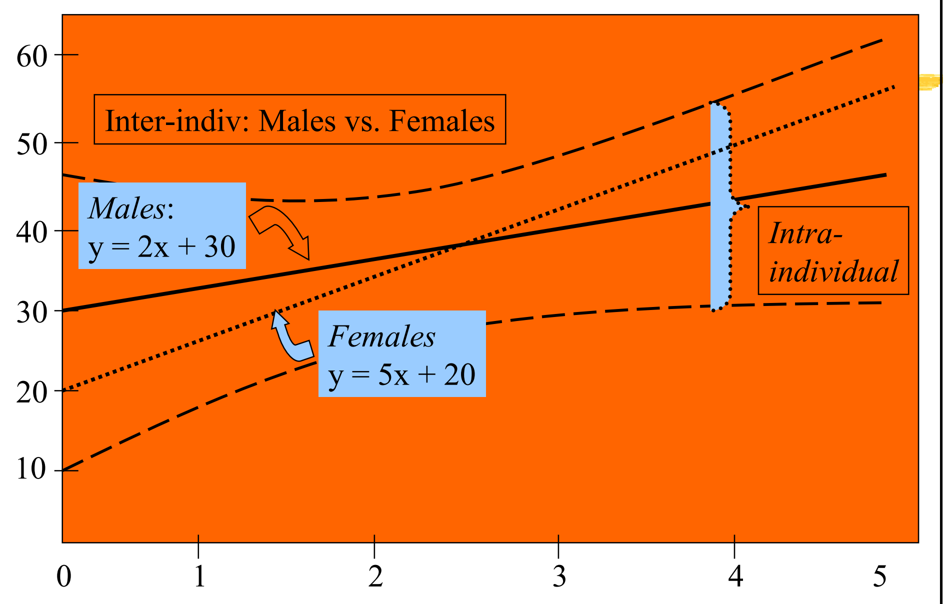
Repeated Measures ANOVA: Inter-individual change Interaction = Group Avg change over time



Intra-individual change:



Multi-level models: Both Inter- and Intra-individual change



Predictors of change

- Invariant: unmodifiable subject or group characteristics, e.g., race, ethnicity, gender, pre-test scores
- Time-varying: predictors that vary over the study period, e.g., drug use, knowledge, maturation, attitudes, etc.

Examples

- Invariant: are change trajectories different for males vs. females?
- Time-varying: does change in mental health status predict change in drug use?

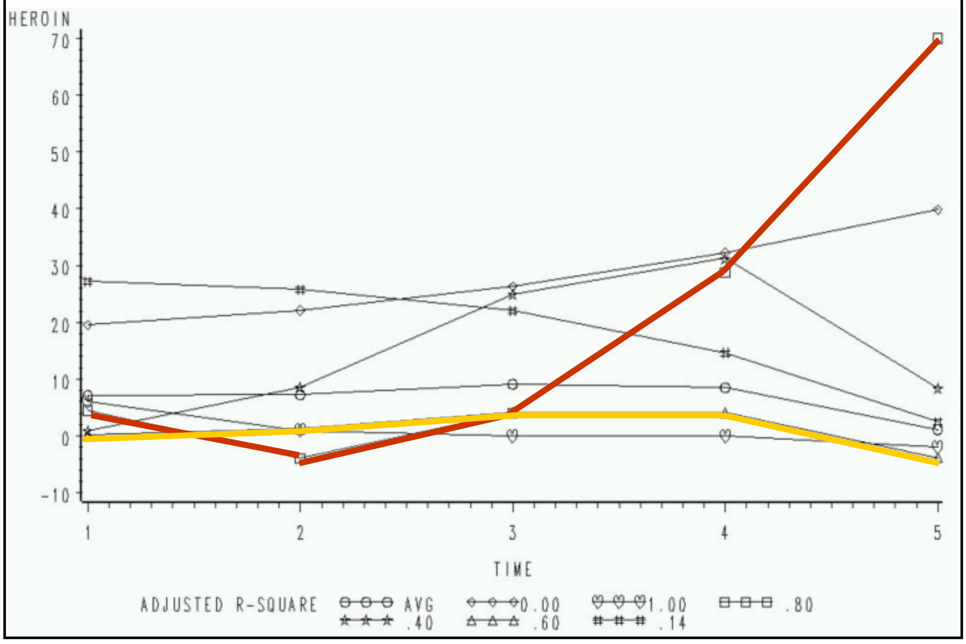
Parameter estimation

- Traditional Analysis of Variance and standard regression procedures: OLS
- Individual growth curve models: ML & REML, Empirical Bayes
- Procedures have different strengths & limitations

Example: Estimating individual growth parameters

- OLS to estimate individual intercepts and slopes:
  - sensitive to extreme values
  - individual parameters not equally reliable (depends on # of observations per person)

OLS estimate of change:



Maximum Likelihood

- Statistical principle, associated with MLMs, SEM

<u>2 methods</u>

- Full Info ML (FIML): Used to estimate mean & dispersion of the DV

OR

- Restricted ML (REML): applied to the residuals (after removing fixed effects); regression coefficients estimated using GLS to weight variance components

OLS vs. FIML vs. REML
(from Kreft & de Leeuw (1998)

- Fixed coefficients (the mean solution)
  - All 3 give unbiased fixed parameter estimates
  - OLS somewhat less efficient in smaller data sets (i.e. smaller number of clusters)
- Variance components (random effects)
  - Unclear which method is best: REML appears less biased but is less precise than FIML

Missing Data

- Balanced vs. unbalanced data
- Balanced requires listwise deletion or replacement of missing values
- Unbalanced allows for missing values
- Statistical models using ML estimation and EM algorithm can handle unbalanced data

Some methods for analysis of change

- Traditional fixed effects models: Repeated Measures ANOVA, Pre/post ANCOVA
- Individual growth models:
  - "slopes-as-outcomes" (a.k.a. 2-step, varying parameters, individual growth curves)
  - multilevel models (e.g., HLM, random coefficients models, mixed linear models)
  - latent growth models (a.k.a. latent variable modeling, covariance structures, causal modeling, structured means)

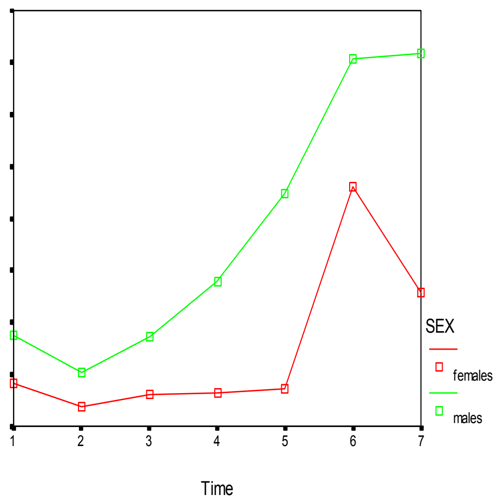
Repeated Measures ANOVA

Repeated Measures ANOVA

- ⌘Model differences in DV as an effect of group, time and group*time
- ⌘ Individual differences (subject*time) part of the error term, i.e."unexplained variance"
- ⌘Output: omnibus test (F-ratio)

Hypothesis testing for RMA

- ⌘Between-subjects: do groups differ?
- ⌘Within-subjects: with the same subjects, are there differences across occasions?
- ⌘Do groups differ at different time points?
- ⌘Is there a significant trend across occasions?



Considerations for choosing Repeated Measures ANOVA

- Continuous DVs
- Change parameters: differences between means, trend analysis (groups only)
- Fixed effects model
- Level of change: Inter-individual
- Parameter estimation: OLS
- Missing data: requires balanced data; usually listwise deletion

Example: Predicting Math Achievement

Between subjects:

Math = SES + gender + advmath8 + gender*advmath8

covariate | main effects | interaction

Within subjects: Math achievement at 8^{th}, 10^{th} & 12^{th} grades for all and by group

* the individual x wave = error term in RMA

Data structure: “wide” format

id	advmath8	gender	achmat08	achmat10	achmat12	ses
1	1	0	47.44	55.23	59.69	23
2	1	1	54.49	53.25	55.36	20
3	0	0	58.05	55.24	56.73	20
4	0	1	43.09	52.18	51.08	29
5	1	0	64.91	58.64	55.75	21

Example: Math Achievement

Within-Subjects Factors

Measure: MEASURE_1

Math	Dependent Variable
1	achmat08
2	achmat10
3	achmat12

Between-Subjects Factors

		Value Label	N
Gender	0	Male	224
	1	Female	267
Advanced Math Taken in Eighth Grade	0	No	265
	1	Yes	226

Between subjects effects: Do groups differ on the outcome?*

Tests of Between-Subjects Effects

Measure: MEASURE_1
Transformed Variable: Average

Source	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	453489.513	1	453489.513	3159.649	.000	.867	3159.649	1.000
gender	2094.662	1	2094.662	14.594	.000	.029	14.594	.968
advmath8	9332.007	1	9332.007	65.020	.000	.118	65.020	1.000
ses	9408.397	1	9408.397	65.552	.000	.119	65.552	1.000
gender * advmath	77.554	1	77.554	.540	.463	.001	.540	.114
Error	69753.284	486	143.525					

a. Computed using alpha = .05

*The outcome for these questions is OVERALL math achievement, i.e., collapsing test scores across all 3 grades.

Estimates

Measure: MEASURE_1

Gender	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Male	58.458[a]	.465	57.544	59.372
Female	56.044[a]	.425	55.209	56.880

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.

Estimates

Measure: MEASURE_1

Advanced Math Taken in Eighth Grade	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
No	54.714[a]	.426	53.876	55.552
Yes	59.788[a]	.463	58.879	60.697

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.

*After controlling for SES, a covariate in the model

Within subject effects: Does the outcome vary over time?*

Estimates

Measure: MEASURE_1

Math	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
1	57.100[a]	.371	56.372	57.828
2	57.352[a]	.318	56.728	57.976
3	57.300[a]	.322	56.667	57.934

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.

*After controlling for SES, a covariate in the model

Tests of Within-Subjects Effects

Measure: MEASURE_1

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Math	Sphericity Assumed	49.932	2	24.966	2.263	.105	.005	4.527	.462
	Greenhouse-Geisser	49.932	1.676	29.800	2.263	.114	.005	3.792	.420
	Huynh-Feldt	49.932	1.695	29.466	2.263	.113	.005	3.835	.422
	Lower-bound	49.932	1.000	49.932	2.263	.133	.005	2.263	.324
Math * gender	Sphericity Assumed	19.794	2	9.897	.897	.408	.002	1.794	.205
	Greenhouse-Geisser	19.794	1.676	11.813	.897	.392	.002	1.503	.191
	Huynh-Feldt	19.794	1.695	11.681	.897	.393	.002	1.520	.192
	Lower-bound	19.794	1.000	19.794	.897	.344	.002	.897	.157
Math * advmath8	Sphericity Assumed	545.915	2	272.958	24.746	.000	.048	49.491	1.000
	Greenhouse-Geisser	545.915	1.676	325.806	24.746	.000	.048	41.463	1.000
	Huynh-Feldt	545.915	1.695	322.159	24.746	.000	.048	41.933	1.000
	Lower-bound	545.915	1.000	545.915	24.746	.000	.048	24.746	.999
Math * ses	Sphericity Assumed	38.058	2	19.029	1.725	.179	.004	3.450	.363
	Greenhouse-Geisser	38.058	1.676	22.713	1.725	.184	.004	2.891	.331
	Huynh-Feldt	38.058	1.695	22.459	1.725	.184	.004	2.923	.333
	Lower-bound	38.058	1.000	38.058	1.725	.190	.004	1.725	.259
Math * gender * advmath8	Sphericity Assumed	87.004	2	43.502	3.944	.020	.008	7.887	.710
	Greenhouse-Geisser	87.004	1.676	51.924	3.944	.026	.008	6.608	.654
	Huynh-Feldt	87.004	1.695	51.343	3.944	.026	.008	6.683	.658
	Lower-bound	87.004	1.000	87.004	3.944	.048	.008	3.944	.509
Error(Math)	Sphericity Assumed	10721.731	972	11.031					
	Greenhouse-Geisser	10721.731	814.335	13.166					
	Huynh-Feldt	10721.731	823.552	13.019					
	Lower-bound	10721.731	486.000	22.061					

a. Computed using alpha = .05

Within subjects: trend analysis*

Tests of Within-Subjects Contrasts

Measure: MEASURE_1

Source	Math	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Math	Linear	44.783	1	44.783	2.964	.086	.006	2.964	.405
	Quadratic	5.149	1	5.149	.741	.390	.002	.741	.138
Math * gender	Linear	19.367	1	19.367	1.282	.258	.003	1.282	.204
	Quadratic	.428	1	.428	.062	.804	.000	.062	.057
Math * advmath	Linear	461.470	1	461.470	30.538	.000	.059	30.538	1.000
	Quadratic	84.445	1	84.445	12.151	.001	.024	12.151	.936
Math * ses	Linear	36.074	1	36.074	2.387	.123	.005	2.387	.338
	Quadratic	1.984	1	1.984	.285	.593	.001	.285	.083
Math * gender advmath8	Linear	62.738	1	62.738	4.152	.042	.008	4.152	.529
	Quadratic	24.265	1	24.265	3.492	.062	.007	3.492	.462
Error(Math)	Linear	7344.148	486	15.111					
	Quadratic	3377.583	486	6.950					

a. Computed using alpha = .05

*3 time points allow for the testing of a linear + quadratic component in traditional fixed effects RMA models

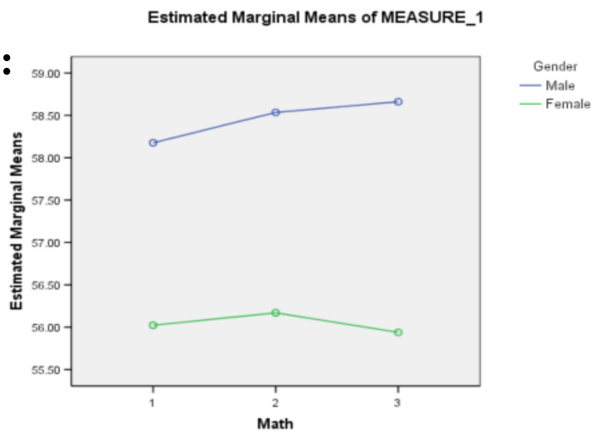
2. Gender * Math

Measure: MEASURE_1

Gender	Math	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Male	1	58.177[a]	.548	57.100	59.254
	2	58.535[a]	.470	57.611	59.458
	3	58.662[a]	.477	57.724	59.599
Female	1	56.023[a]	.501	55.039	57.008
	2	56.170[a]	.430	55.326	57.014
	3	55.939[a]	.436	55.082	56.796

a. Covariates ap values: Socio

Within-subjects: Do groups vary differently across grades?



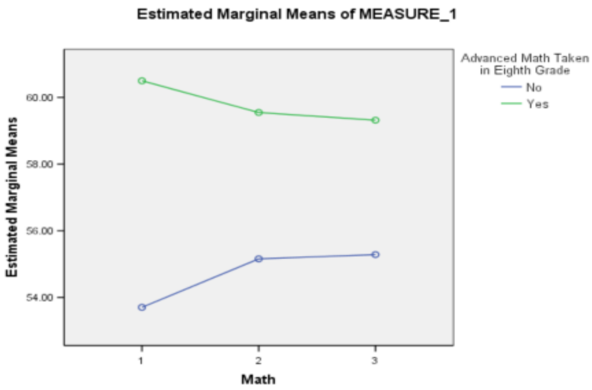
3. Advanced Math Taken in Eighth Grade * Math

Measure: MEASURE_1

Advanced Math Taken in Eighth Grade	Math	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
No	1	53.702[a]	.502	52.714	54.689
	2	55.156[a]	.431	54.310	56.003
	3	55.284[a]	.437	54.425	56.143
Yes	1	60.499[a]	.545	59.428	61.570
	2	59.548[a]	.467	58.630	60.466
	3	59.317[a]	.474	58.385	60.249

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.

Within-subjects: Do groups vary differently across grades?



Within subject: Do groups vary differently across grades?

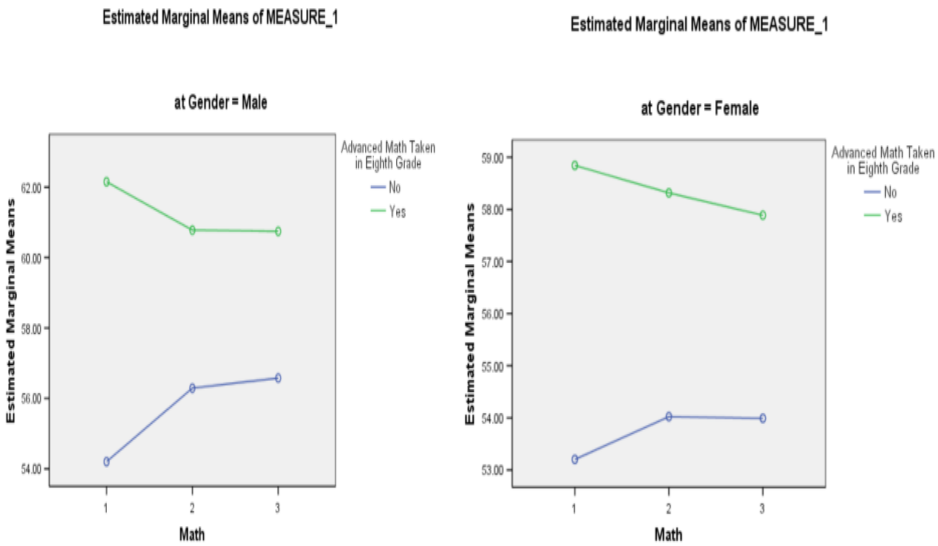
Advmath8*gender*math

4. Advanced Math Taken in Eighth Grade * Gender * Math

Measure: MEASURE_1

Advanced Math Taken in Eighth Grade	Gender	Math	Mean	Std. Error	95% Confidence Interval Lower Bound	Upper Bound
No	Male	1	54.201[a]	.738	52.751	55.651
		2	56.291[a]	.633	55.047	57.534
		3	56.577[a]	.642	55.315	57.839
	Female	1	53.202[a]	.682	51.861	54.543
		2	54.022[a]	.585	52.873	55.172
		3	53.991[a]	.594	52.824	55.157
Yes	Male	1	62.153[a]	.810	60.562	63.744
		2	60.779[a]	.694	59.414	62.143
		3	60.746[a]	.705	59.361	62.131
	Female	1	58.844[a]	.733	57.405	60.284
		2	58.317[a]	.628	57.083	59.551
		3	57.888[a]	.638	56.635	59.141

a. Covariates appearing in the model are evaluated at the following values: Socio-Economic Status = 18.42.



Conclusions about change in math achievement using RMA: Within-subjects hypotheses

- After controlling for SES:
  - overall math scores do not differ significantly across grades $F(2, 485) = 1.57, p > .05$
  - Math scores do not vary significantly across grades for males vs. females $F(2, 485) = .79, p > .05$
  - Math scores do vary significantly across grades for
    - Those who did vs. did not take advanced math in 8^{th} grade $F(2, 485) = 17.52, p < .05$
    - Males & females, depending on whether they took advanced math in 8^{th} grade or not $F(2, 485) = 3.04, p < .05$
  - Math scores show a significant linear & quadratic trend for
    - Adv math in 8^{th} grade: $F(1,486) = 30.54$ (linear) & $F(1,486) = 12.15$ (quadratic)
    - Gender x advmath8: $F(1,486) = 4.15$ (linear) & $F(1,486) = 3.49$ (quad)

Conclusions about change in math achievement using RMA: Between-subjects hypotheses

- Overall (collapsing grades 8, 10 & 12 scores):
  - SES is significantly related to math scores $F(1, 486) = 65.55$, $p < .05$
  - Males & females differ in math scores $F(1, 486) = 14.59$, $p < .05$
  - Those who took advanced math in 8^{th} grade differ from those who did not $F(1, 486) = 65.02$, $p < .05$
  - The effect of gender does not depend on whether one took advanced math in 8^{th} grade

Pre/Posttest ANCOVA

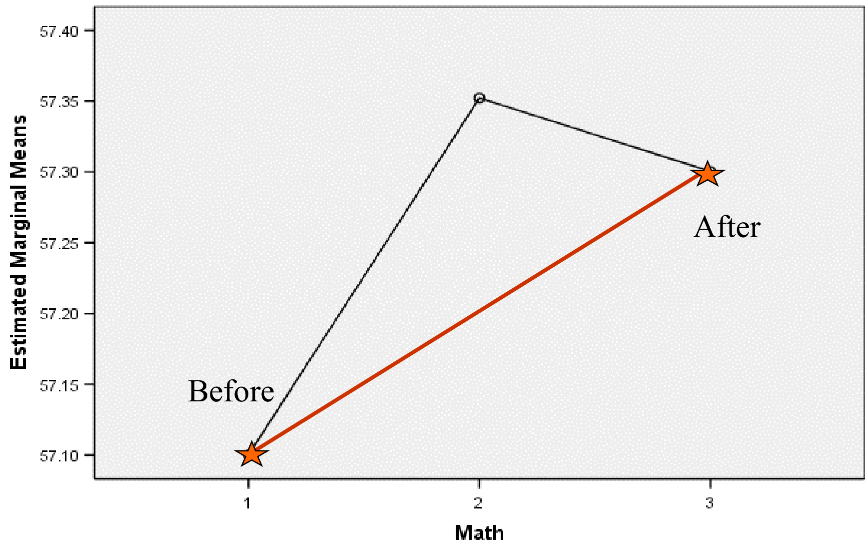
Pre/Post ANCOVA

- DV = difference or gain score
- Covariates include pre-test measure
- Difference from pre to post is function of covariates and main effects
- Output: omnibus test (F-ratio) of overall model, significance tests for each predictor
- Predictors assumed to have equal effect on DV for each individual
  - Fixed coefficient: slope
  - Varying coefficient: intercept

Hypothesis testing for Pre/Posttest ANCOVA

- Between subjects
  - Do groups differ with respect to gain scores, taking into consideration different starting points (e.g., 8th grade math score)?
  - What predicts the gain score after accounting for baseline status (e.g., 8th grade math score)

Outcome for Pre/Post ANCOVA: Gain score (12^{th} grade minus 8^{th} grade score)



Considerations for choosing pre/post ANCOVA

- Continuous DV
- Change parameters: none...change reflected in DV
- Fixed effects model
- Level of change: Inter-individual
- Parameter estimation: OLS
- Missing data: requires balanced data; listwise deletion is default

Example: Predicting Gains in Math

Between subjects:

Gain = achmat08+SES+gender+advmath8+gender*advmath8

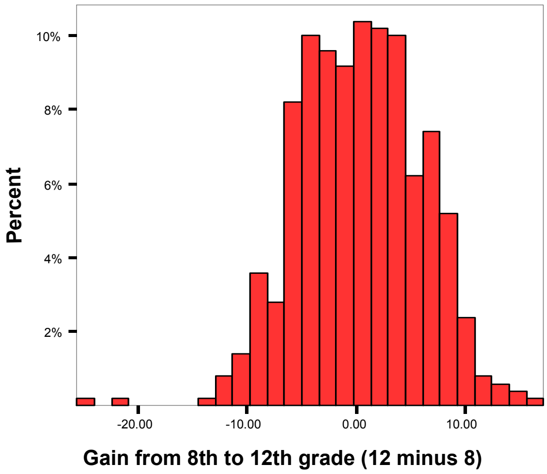
covariates | main effects | interaction

Within subjects: error term

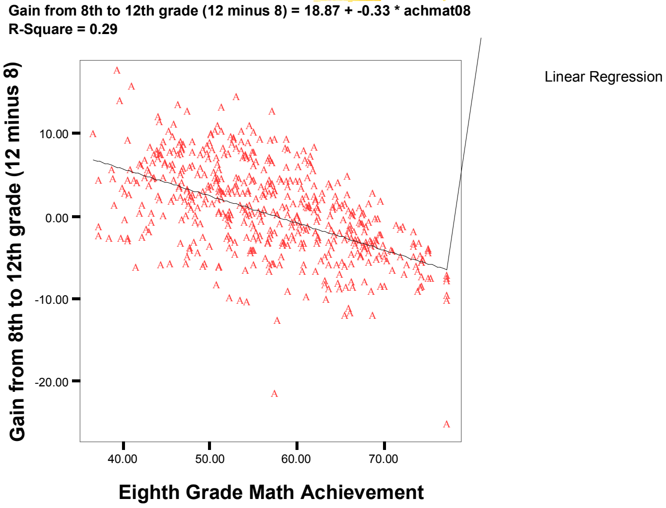
Data structure: "wide" format

id	advmath8	gender	achmat08	achmat12	Math_dif	ses
1	1	0	47.44	59.69	12.25	23
2	1	1	54.49	55.36	.87	20
3	0	0	58.05	56.73	-1.32	20
4	0	1	43.09	51.08	7.99	29
5	1	0	64.91	55.75	-9.16	21

Distribution of gain scores



Correlation of starting point & gain



Between-subjects

Between-Subjects Factors

		Value Label	N
Gender	0	Male	224
	1	Female	267
Advanced Math Taken in Eighth Grade	0	No	265
	1	Yes	226

Descriptive Statistics

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

Gender	Advanced Math Taken	Mean	Std. Deviation	N
Male	No	2.3614	5.48331	122
	Yes	-1.4770	5.82381	102
	Total	.6136	5.94524	224
Female	No	.8233	5.30488	143
	Yes	-.9248	5.48513	124
	Total	.0114	5.44961	267
Total	No	1.5314	5.43207	265
	Yes	-1.1740	5.63454	226
	Total	.2861	5.68313	491

Testing between subject effects

Tests of Between-Subjects Effects

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

Source	Type I Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Corrected Model	5065.454[b]	5	1013.091	45.662	.000	.320	228.310	1.000
Intercept	40.198	1	40.198	1.812	.179	.004	1.812	.269
achmat08	4603.311	1	4603.311	207.480	.000	.300	207.480	1.000
ses	173.922	1	173.922	7.839	.005	.016	7.839	.798
gender	231.910	1	231.910	10.453	.001	.021	10.453	.897
advmath8	10.661	1	10.661	.481	.489	.001	.481	.106
gender * advmath8	45.651	1	45.651	2.058	.152	.004	2.058	.299
Error	10760.575	485	22.187					
Total	15866.228	491						
Corrected Total	15826.029	490						

a. Computed using alpha = .05

b. R Squared = .320 (Adjusted R Squared = .313)

Parameters

Parameter Estimates

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Intercept	18.028	1.518	11.873	.000	15.044	21.011	.225	11.873	1.000
achmat08	-.349	.026	-13.305	.000	-.400	-.297	.267	13.305	1.000
ses	.084	.033	2.558	.011	.019	.148	.013	2.558	.723
[gender=0]	.704	.638	1.103	.271	-.550	1.957	.003	1.103	.196
[gender=1]	0[b]	.	.	.	.	.	.	.	.
[advmath8=0]	-.223	.597	-.374	.709	-1.395	.949	.000	.374	.066
[advmath8=1]	0[b]	.	.	.	.	.	.	.	.
[gender=0] * [advmath8=0]	1.232	.859	1.434	.152	-.456	2.920	.004	1.434	.299
[gender=0] * [advmath8=1]	0[b]	.	.	.	.	.	.	.	.
[gender=1] * [advmath8=0]	0[b]	.	.	.	.	.	.	.	.
[gender=1] * [advmath8=1]	0[b]	.	.	.	.	.	.	.	.

a. Computed using alpha = .05

b. This parameter is set to zero because it is redundant.

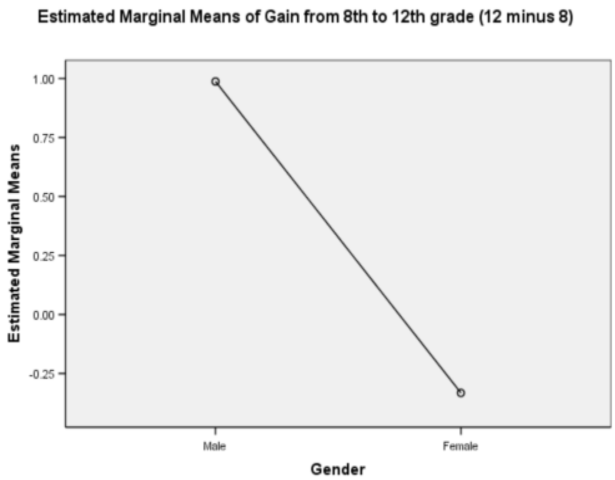
Univariate Tests

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Contrast	205.200	1	205.200	9.249	.002	.019	9.249	.859
Error	10760.575	485	22.187					

The F tests the effect of Gender. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Computed using alpha = .05



Do males & females differ in math gains from 8th to 12th grade?

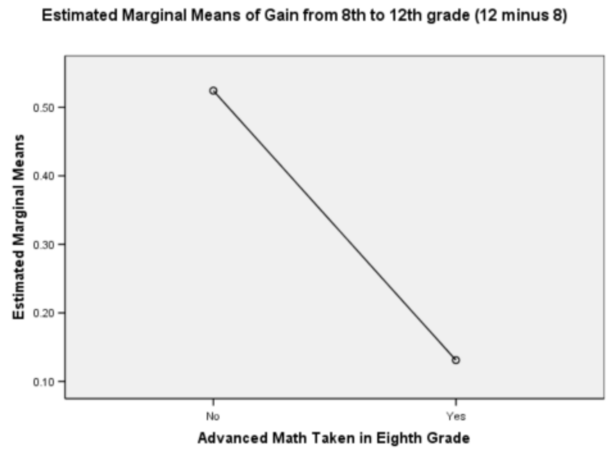
Univariate Tests

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[a]
Contrast	15.916	1	15.916	.717	.397	.001	.717	.135
Error	10760.575	485	22.187					

The F tests the effect of Advanced Math Taken in Eighth Grade. This test is based on the linearly independent pairwise comparisons among the estimated marginal means.

a. Computed using alpha = .05



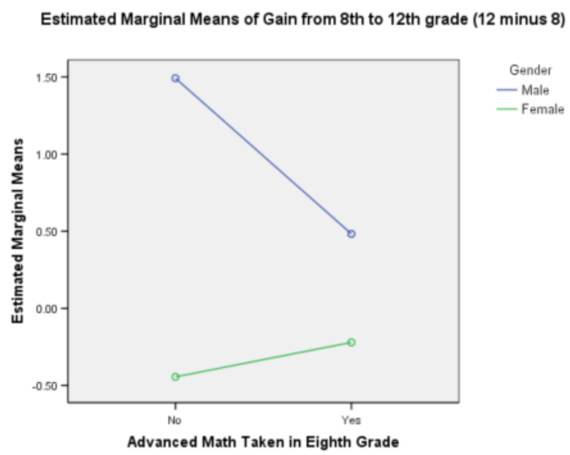
Do those who took advanced math in 8^{th} grade differ in math gains from 8^{th} to 12^{th} grade?

4. Gender * Advanced Math Taken in Eighth Grade

Dependent Variable: Gain from 8th to 12th grade (12 minus 8)

Gender	Advanced Math Taken in Eighth Grade	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Male	No	1.492[a]	.432	.644	2.340
	Yes	.483[a]	.489	-.478	1.444
Female	No	-.444[a]	.405	-1.240	.352
	Yes	-.221[a]	.427	-1.060	.618

a. Covariates appearing in the model are evaluated at the following values: Eighth



Does the effect of gender on math gains from 8^{th} to 12^{th} grade depend on whether advanced math was taken in 8^{th} grade?

Test of simple effects: gender x advmath8

Cell Means and Standard Deviations
DV: Math_dif Gain from 8th to 12th grade (12 minus 8)

FACTOR	CODE	Mean	Std. Dev.	N
gender	Male			
advmath8	No	2.361	5.483	122
advmath8	Yes	-1.477	5.824	102
gender	Female			
advmath8	No	.823	5.305	143
advmath8	Yes	-.925	5.485	124
For entire sample		.286	5.683	491

Tests of simple effects: gender x advmath8

Tests of Significance for Math_dif using UNIQUE sums of squares

Source of Variation	SS	DF	MS	F	Sig of F
WITHIN+RESIDUAL	14789.85	488	30.31		
ADVMATH8 WITHIN Male	842.70	1	842.70	27.81	.000
ADVMATH8 WITHIN Female	196.05	1	196.05	6.47	.011
(Model)	1036.18	2	518.09	17.09	.000
(Total)	15826.03	490	32.30		

R-Squared = .065
Adjusted R-Squared = .062

Conclusions about change in math achievement using Pre-post ANCOVA

- 8^{th} grade math achievement is significantly & negatively related to gains in math from 8^{th} to 12^{th} grade
  - Lower 8^{th} grade scores show greater gains (or, higher scores show less gains)
- SES is significantly & positively related to gains in math from 8^{th} to 12^{th} grade
  - One unit increase in SES scale = .08 increase in gain score

Conclusions about change in math achievement using Pre-post ANCOVA

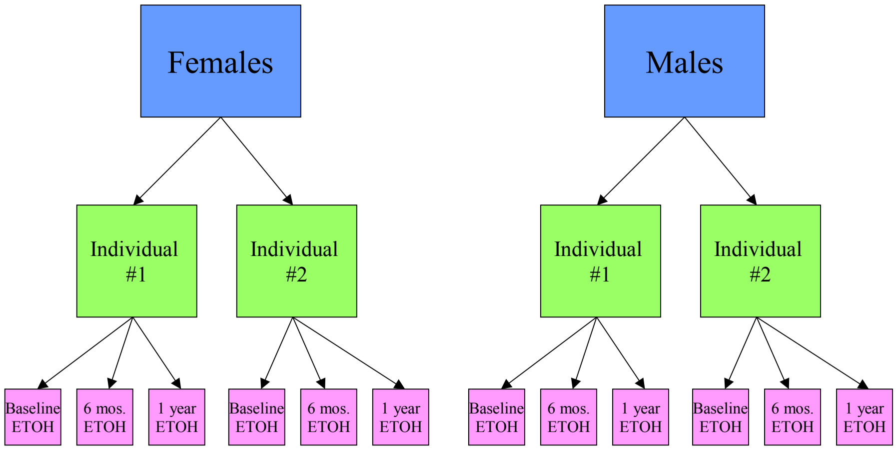
- On average, males show significantly greater gains than females
  - Males on average show positive gains, while females on average show a decrement in scores
- Average male and female gains depend on whether advanced math was taken in 8^{th} grade
  - Males who did NOT take advanced math showed the greatest average gains, significantly greater than males who did take advanced math
  - Females who did NOT take advanced math on average showed significantly greater <u>loss</u> in math scores than females who did

Multilevel models of individual growth curves

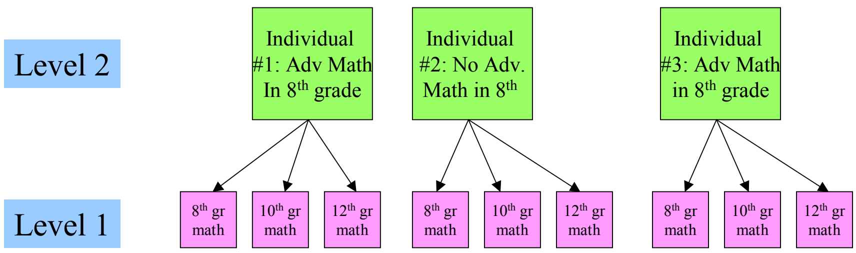
Also known as...

- Random coefficients models
- Contextual effects models
- Multilevel mixed effects models
- Random parameter models
- Full contextual models
- Variance components models
- Multilevel linear models
- Hierarchical linear models (HLMs)

Multilevel Structure of Longitudinal Data



Multilevel Structure of Longitudinal Data



Level 1: each gets own trajectory; intra-individual change
Level 2: predictors of change; inter-individual change

Questions about change

- What is the average change trajectory?
- Is there significant individual variation in initial status (baseline) and/or change?
- What variables account for the between-person variation in initial status and change?

Data structure: "Long" format

ID	advmath8	gender	SES	Grade	Mathach	Time
1	1	0	23	8	47.44	1
1	1	0	23	10	55.23	3
1	1	0	23	12	59.69	5
2	1	1	20	8	54.49	1
2	1	1	20	10	53.25	3
2	1	1	20	12	55.36	5
3	0	0	20	8	58.05	1
3	0	0	20	10	55.24	3
3	0	0	20	12	56.73	5

Early attempts at multilevel analysis: "Slopes as outcomes" approach

2-step models:

- Step 1: Regression equation for each individual → *Regress DV on time*
- Step 2: Regression equation for whole sample → *Regress slopes (new DV) on predictors of change*
- Step 2 can be done with intercept as well

Change Parameters

Step 1: **Math achievement = time**

Output: α, β_1 (& R^2) for each person

* α = baseline math; β_1 = rate of change in math; R^2 = fit of growth model for each person

Step 2: $\beta_1 = \alpha$ **+ gender + advmath8 + SES**

and/or

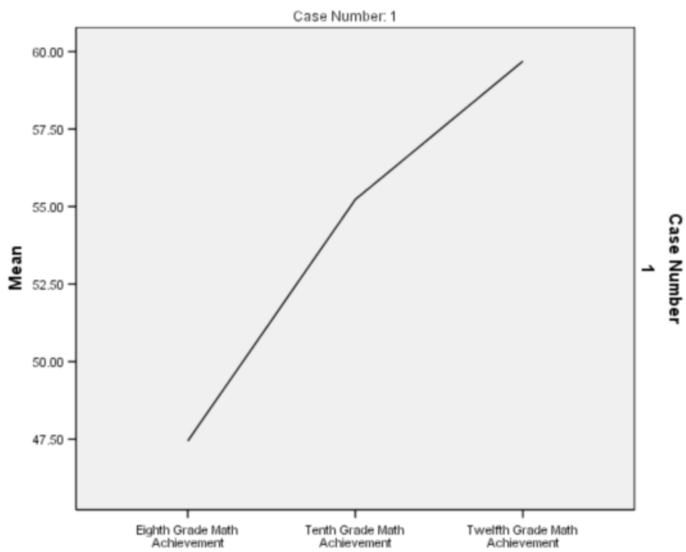
α = **gender + SES + homecomputer**

Model Summary

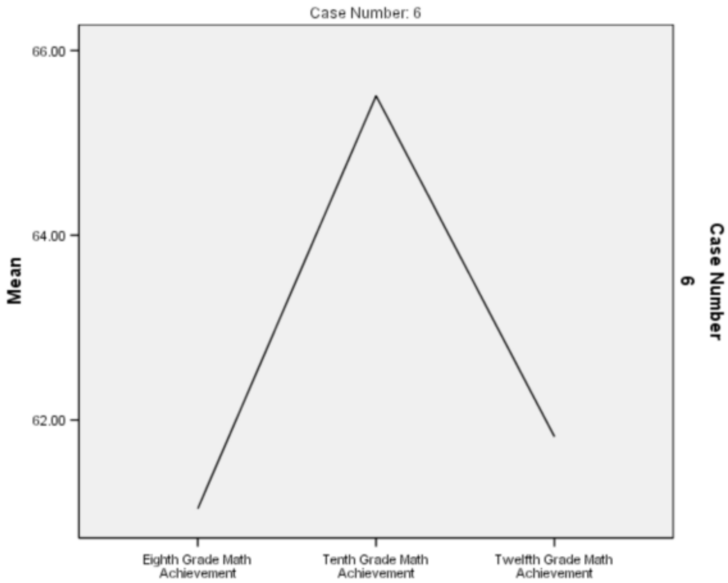
id1	Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	1	.988(a)	**.976**	.952	1.35947
2	1	.410(a)	.168	-.663	1.36763
3	1	.469(a)	.220	-.559	1.75547
4	1	.805(a)	.648	.297	4.16005
5	1	.978(a)	.957	.913	1.37988
6	1	.163(a)	**.027**	-.947	3.33131
7	1	1.000(a)	.999	.999	.06124
8	1	.780(a)	.608	.215	3.56809
9	1	.554(a)	**.307**	-.385	.05307
10	1	.781(a)	.609	.219	1.17167

a Predictors: (Constant), Time

$R^2 = .98$

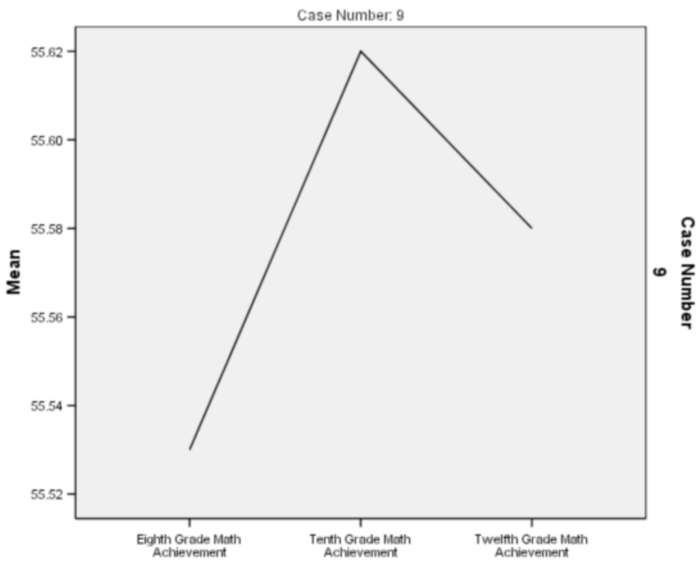


$R^2 = .03$



Case Number: 6
66.00
64.00
62.00
Mean
Case Number
6
Eighth Grade Math Achievement
Tenth Grade Math Achievement
Twelfth Grade Math Achievement

$R^2 = .31$



Case Number: 9
55.62
55.60
55.58
55.56
55.54
55.52
Mean
Case Number
9
Eighth Grade Math Achievement
Tenth Grade Math Achievement
Twelfth Grade Math Achievement

id1	Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
			B	Std. Error	Beta		
1	1	(Constant)	44.933	1.642		27.369	.023
		Time	3.063	.481	.988	6.372	.099
2	1	(Constant)	53.714	1.652		32.523	.020
		Time	.218	.484	.410	.450	.731
3	1	(Constant)	57.663	2.120		27.201	.023
		Time	-.330	.621	-.469	-.532	.689
4	1	(Constant)	42.791	5.024		8.518	.074
		Time	1.998	1.471	.805	1.358	.404
5	1	(Constant)	66.637	1.666		39.989	.016
		Time	-2.290	.488	-.978	-4.694	.134
6	1	(Constant)	62.205	4.023		15.463	.041
		Time	.195	1.178	.163	.166	.896
7	1	(Constant)	67.493	.074		912.665	.001
		Time	-.898	.022	-1.000	-41.454	.015
8	1	(Constant)	58.047	4.309		13.471	.047
		Time	-1.570	1.262	-.780	-1.245	.431
9	1	(Constant)	55.539	.064		866.569	.001
		Time	.012	.019	.554	.666	.626
10	1	(Constant)	58.891	1.415		41.621	.015
		Time	.518	.414	.781	1.249	.430

Step 2: predicting slope or rate of change

Tests of Between-Subjects Effects

Dependent Variable: math ach estimated linear slope

Source	Type I Sum of Squares	df	Mean Square	F	Sig.	Noncent. Parameter	Observed Power[a]
Corrected Model	418.251[b]	5	83.650	71.067	.000	355.335	1.000
Intercept	2.512	1	2.512	2.134	.145	2.134	.308
baseline	385.576	1	385.576	327.575	.000	327.575	1.000
ses	15.968	1	15.968	13.566	.000	13.566	.957
gender	14.463	1	14.463	12.288	.000	12.288	.938
advmath8	.061	1	.061	.052	.820	.052	.056
gender * advmath	2.183	1	2.183	1.855	.174	1.855	.274
Error	570.875	485	1.177				
Total	991.639	491					
Corrected Total	989.127	490					

a. Computed using alpha = .05

b. R Squared = .423 (Adjusted R Squared = .417)

Note: “Baseline” = estimated intercept

Gender: mean slope differences

Estimates

Dependent Variable: math ach estimated linear slope

Gender	Mean	Std. Error	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Male	.255[a]	.073	.110	.399
Female	-.082[a]	.067	-.213	.049

a. Covariates appearing in the model are evaluated at the following values: math ach estimated intercept = 56.7286, Socio-Economic Status = 18.42.

Pairwise Comparisons

Dependent Variable: math ach estimated linear slope

(I) Gender	(J) Gender	Mean Difference (I-J)	Std. Error	Sig.[a]	95% Confidence Interval for Difference[a] Lower Bound	95% Confidence Interval for Difference[a] Upper Bound
Male	Female	.337*	.100	.001	.141	.532
Female	Male	-.337*	.100	.001	-.532	-.141

Based on estimated marginal means

*. The mean difference is significant at the .05 level.

a. Adjustment for multiple comparisons: Bonferroni.

Limitations to slopes-as-outcomes

- Growth parameters not of equal stability across individuals
  - Vary in number of observations
  - Model doesn't fit for each person (see R^2)
- Fails to take into account context effects
  - If individuals are nested within groups, context can violate assumption of independence (ICCs are non-zero)

“Borrowing strength”

- Multilevel models use observed data (like OLS) from the individual as well as others in the data set
  - Missing data per individual not as much an issue
  - Extreme values not as much an issue
- Also uses an “expectation” or model-based approach to weight estimates
  - “Shrinks” extreme values toward the mean

Multilevel Models

- Multilevel data: repeated measures nested within persons
- Steps 1 and 2 are simultaneous
- Find overall solution (i.e. average growth) and assess deviations (individual differences)
- Tests predictors of change as well as variance in the change parameters

Step 1 and 2 conceptual formulas:

Step 1:

$$Y_{ij} = \pi_{0j} + \pi_{1j}(\mathrm{TIME})_{ij} + r_{ij}$$

where

$\mathbf{Y_{ij}}$ = outcome for *j*th individual at *i*th time point
$\boldsymbol{\pi_{0j}}$ and $\boldsymbol{\pi_{1j}}$ = individual growth parameters (intercept and slope) for the *j*th person
$\mathbf{r_{ij}}$ = within-person residual for *j*th person at *i*th time point.

Step 2:

$$\pi_{0j} = \beta_{00} + \beta_{01}\mathit{PREDICTOR}_j + u_{0j}$$
$$\pi_{1j} = \beta_{10} + \beta_{11}\mathit{PREDICTOR}_j + u_{1j}$$

where

$\boldsymbol{\beta_{00}}$ and $\boldsymbol{\beta_{10}}$ = average intercept and slope
$\mathbf{u_{0j}}$ and $\mathbf{u_{1j}}$ = between-persons residual for intercept and slope

Steps 1 & 2 combined
mixed effects model

Fixed effects

$$Y_{ij} = [\beta_{00} + \beta_{10}(\mathrm{TIME})_{ij} + \beta_{01}\mathit{PREDICTOR}_j + \beta_{11}\mathit{PREDICTOR}_j (\mathrm{TIME})_{ij}]$$

$$+ [u_{0j} + u_{1j}(\mathrm{TIME})_{ij} + r_{ij}]$$

Random effects

Considerations for choosing multilevel models

- Continuous (equal interval or ordinal) or dichotomous
- Change parameters: mean intercept and slope, variances of each
- Effects: fixed and random (i.e. mixed)
- Level of change: Inter- & intra-individual
- Parameter estimation: Some form of EB/ML
- Missing data: allows unbalanced data

Parameterizing time

- Data in LONG format
- Level 1 model: "unconditional growth model" → outcome is a function of time

$$Y_{ij} = [\beta_{00} + \beta_{10}\,(\mathrm{TIME})_{ij}] + [u_{0j} + u_{1j}(\mathrm{TIME})_{ij} + r_{ij}]$$

- How to code time? What is "true" baseline?
  - Estimated vs. observed baseline

Coding time

id	advmath8	gender	SES	Grade	Mathach	Time
1	1	0	23	8	47.44	1
1	1	0	23	10	55.23	3
1	1	0	23	12	59.69	5
2	1	1	20	8	54.49	1
2	1	1	20	10	53.25	3
2	1	1	20	12	55.36	5
3	0	0	20	8	58.05	1
3	0	0	20	10	55.24	3

Step 1: Unconditional model Fixed effects (avg change)

Type I Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	499.000	25836.725	.000
Time	1	499.000	1.550	.214

a. Dependent Variable: Math achievement.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	56.592427	.439584	499.000	128.741	.000	55.728763	57.456090
Time	.078900	.063365	499.000	1.245	.214	-.045596	.203396

a. Dependent Variable: Math achievement.

Step 1: Unconditional model
Random effects (variability in change)

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		7.248950	.458464	15.811	.000	6.403839	8.205590
Intercept + Time [subject = id]	UN (1,1)	86.045707	6.153153	13.984	.000	74.792756	98.991721
	UN (2,1)	-5.979332	.754898	-7.921	.000	-7.458906	-4.499758
	UN (2,2)	1.101464	.139420	7.900	.000	.859464	1.411605

a. Dependent Variable: Math achievement.

Statistical goals:

1. *reduce the residual variance by adding Predictors/explanatory variables into the model*
2. *Explain variance in initial status (UN(1,1)) and/or rate of change (UN (2,2)) by adding predictors.*

Model fit statistics

Information Criteria[a]

-2 Restricted Log Likelihood	9224.935
Akaike's Information Criterion (AIC)	9232.935
Hurvich and Tsai's Criterion (AICC)	9232.962
Bozdogan's Criterion (CAIC)	9258.182
Schwarz's Bayesian Criterion (BIC)	9254.182

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Math achievement.

Statistical goal: improve model fit by adding explanatory variables into the model. Test deviance statistic (-2LL) and residual dfs using Chi Square

nension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	Time	1		1	
Random Effects	Intercept + Time [a]	2	Unstructured	3	id
Residual				1	
Total		4		6	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may yield results that differ from those produced by prior versions. If you are using SPSS 11 syntax, please consult the current syntax reference guide for more information.

b. Dependent Variable: Math achievement.

Step by step model testing: Adding variables & testing fit

1. Add SES (fixed & random)
2. Add gender (fixed)
3. Add gender*time (fixed)
4. Add advmath8 (fixed)
5. Add advmath8*time (fixed)
6. Add gender*advmath8*time (fixed)

Example: Model 1→ add SES

Model Dimension[b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	Time	1		1	
	ses	1		1	
Random Effects	Intercept + Time[a]	2	Unstructured	3	id
Residual				1	
Total		5		7	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may yield results that differ from those produced by prior versions. If you are using SPSS 11 syntax, please consult the current syntax reference guide for more information.

b. Dependent Variable: Math achievement.

Information Criteria[a]

-2 Restricted Log Likelihood	9168.694
Akaike's Information Criterion (AIC)	9176.694
Hurvich and Tsai's Criterion (AICC)	9176.721
Bozdogan's Criterion (CAIC)	9201.939
Schwarz's Bayesian Criterion (BIC)	9197.939

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Math achievement.

-2LL changed from 9224.94, dfs = 4 to 9168.69, dfs = 5

So $\chi 2$ = 9224.94 – 9168.69 = **56.25**

Residual dfs = 5 – 4 = **1**

Significant improvement in model fit, therefore adding SES improves the model

What does SES add to the model?

Estimates of Covariance Parameters [a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		7.248950	.458464	15.811	.000	6.403839	8.205590
Intercept + Time [subject = id]	UN (1,1)	77.065889	5.597783	13.767	.000	66.839631	88.856733
	UN (2,1)	-5.703484	.724890	-7.868	.000	-7.124242	-4.282727
	UN (2,2)	1.101464	.139420	7.900	.000	.859464	1.411605

a. Dependent Variable: Math achievement.

<u>Compare to unconditional model</u>:
-Residual variance (7.25) unchanged: SES is Level 2 variable
-Initial status variance changed by 10% → (86.05-77.06) / 86.05
-Rate of change variance unchanged → 1.10 in both models

Therefore SES explains 10% of the explainable variance in initial status but NOT in rate of change.

The final (best fitting) model

Model Dimension [b]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
	Time	1		1	
	centrSES	1		1	
	gender	2		1	
	advmath8	2		1	
	Time(advmath8)	2		1	
Random Effects	Intercept + Time [a]	2	Unstructured	3	id
Residual				1	
Total		11		10	

a. As of version 11.5, the syntax rules for the RANDOM subcommand have changed. Your command syntax may yield results that differ from those produced by prior versions. If you are using SPSS 11 syntax, please consult the current syntax reference guide for more information.

Information Criteria [a]

-2 Restricted Log Likelihood	8908.403
Akaike's Information Criterion (AIC)	8916.403
Hurvich and Tsai's Criterion (AICC)	8916.430
Bozdogan's Criterion (CAIC)	8941.566
Schwarz's Bayesian Criterion (BIC)	8937.566

The information criteria are displayed in smaller-is-better forms.

a. Dependent Variable: Math achievement.

Deviance statistic (-2LL): $\chi 2 = 316.54$, df = 7
<u>Improved model fit</u>

Change parameters

Estimates of Fixed Effects[b]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	59.398125	.639965	551.319	92.815	.000	58.141057	60.655193
Time	-.293507	.091899	489.000	-3.194	.001	-.474071	-.112942
centrSES	.356427	.044714	487.000	7.971	.000	.268571	.444283
[gender=0]	2.517636	.619641	487.000	4.063	.000	1.300136	3.735135
[gender=1]	0[a]	0	.	.	.	.	.
[advmath8=0]	-7.068674	.784402	486.688	-9.012	.000	-8.609907	-5.527441
[advmath8=1]	0[a]	0	.	.	.	.	.
Time([advmath8=0])	.676356	.125091	489.000	5.407	.000	.430573	.922138
Time([advmath8=1])	0[a]	0	.	.	.	.	.

a. This parameter is set to zero because it is redundant.

b. Dependent Variable: Math achievement.

Change parameters cont'd

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Residual		7.118585	.454326	15.668	.000	6.281564	8.067139
Intercept + Time [subject = id]	UN (1,1)	64.607504	4.854059	13.310	.000	55.761045	74.857448
	UN (2,1)	-4.724575	.659532	-7.164	.000	-6.017234	-3.431916
	UN (2,2)	1.018824	.134628	7.568	.000	.786359	1.320010

a. Dependent Variable: Math achievement.

Residual variance decreased by 2%
25% of explainable variance in initial status explained by model
8% of explainable variance in rate of change explained by model

What can we say about change from these models?

From unconditional growth model:

- On avg, students begin with a score of 56.59 and increased by .08 points per year (the fixed parameters)
- There is significant variability in initial status and rate of change in math achievement (random coefficients)
- Initial status and rate of change are significantly & negatively correlated (random coefficients)

What more can be said about change from these models:

- SES explains 10% of variance in initial status, 0% of rate of change
- Gender explains 0.7% of initial status, 0% of rate of change
- Gender*time adds nothing to the model (worse model fit)
- Advmath8 explains 13.5% of variance in initial status, 0% rate of change
- Advmath8*time explains 2% of variance in initial status, 8% of rate of change; reduces residual by 2%
- Time*gender*advmath8 adds nothing to the model

And more…

- Males taking no adv math in 8th grade & of average SES start with score of 59.4 and lose -.30 points per year
  - SES: higher SES is positively related to initial status (baseline math achievement)
  - Gender: males of avg SES who don't take adv 8th grade math start off 2.52 points lower than similar females
  - Advmath8: students of avg SES taking adv 8th grade math start off 7.07 points lower (baseline)
  - Advmath8*time: on avg., those taking adv math in 8th grade have a more positive rate of change (slope) than those who do not

Improvements over traditional fixed effects models

- Emphasis on individual growth
  - Ability to model both inter- and intra-individual growth
- Ability to model within-person covariance structure (CS, UN, AR1, etc.)
  - RMA assumes sphericity
- Application of models with more than 2 levels

Further issues in multilevel modeling of longitudinal data
(from Bijleveld et al., 1998)

- Time-varying covariates
- Centering of Level1 explanatory variables (i.e. Time) – How to interpret value = 0?
  - Level 2 model: usually center on grand mean, e.g., SES – 18.34
- Metric of outcome variable: same metric at each repeated occasion!

Missing Data

- Deletion methods: default for most stats packages for traditional fixed effects models
- Model-based methods: e.g., ML using EM algorithm – default for many stats packages for multilevel models
- Multiple imputation: separate software, e.g. "MICE" in R

Characteristics of Several Longitudinal Data Analysis Methods

Models	DV	IVs (time varying vs. invariant)	Effects	Level of Change	Parameter Estimation	Unbalanced data[b]
Pre/post ANCOVA	Continuous	Invariant	Fixed	Inter-individual	OLS	No
Repeated Meas ANOVA	Continuous	Invariant	Fixed	Inter-individual	OLS	No
"Slopes-as-Outcomes" regression	Continuous	Time varying & invariant	Fixed	Intra- and Inter-individual	OLS	Yes
Multilevel models	Continuous, & Categorical	Time varying & invariant	Fixed & Random	Intra- and Inter-individual	ML, REML, MML, EB/ML & more[a]	Yes

[a]REML = Restricted Maximum Likelihood, used in e.g., SAS Proc Mixed. MML = Marginal Maximum Likelihood, used in e.g., MIXOR for mixed-effects ordinal probit and logistic regression models. EB/ML = Empirical Bayes/ Maximum Likelihood, used in e.g., MIXREG for mixed-effects regression models. Parameter estimation procedures for multilevel models depend on the statistical package.
[b]Unbalanced data = data in which subjects are not required to have the same number of observations or to be observed at the same time points

Choosing "the" right method

- What is the purpose of the study?
- What is it you hope to find out about change?
- What kind of data do you have?
- Do the assumptions upon which the statistical method is based fit your design and your theory?
- Methods can compliment and/or supplement each other